Impact of Decentralized Learning on Player Utilities in Stackelberg Games

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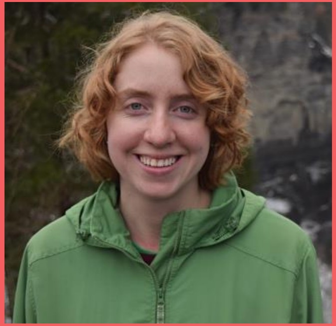
Joint work with Kate Donahue (Cornell), Nicole Immorlica, Brendan Lucier, Alex Slivkins (MSR)




Published at ICML 2024
Presented at ESIF-AIML 2024

High-level overview of this work

We study **Stackelberg games with decentralized learning**.

Captures systems of two sequential, misaligned agents that learn over time

How do the learning dynamics behave?

Outline for the talk

1. Motivation and conceptual overview

2. Model of Stackelberg games with decentralized learning

3. Our analysis of learning dynamics

Motivation: User-AI Interactions

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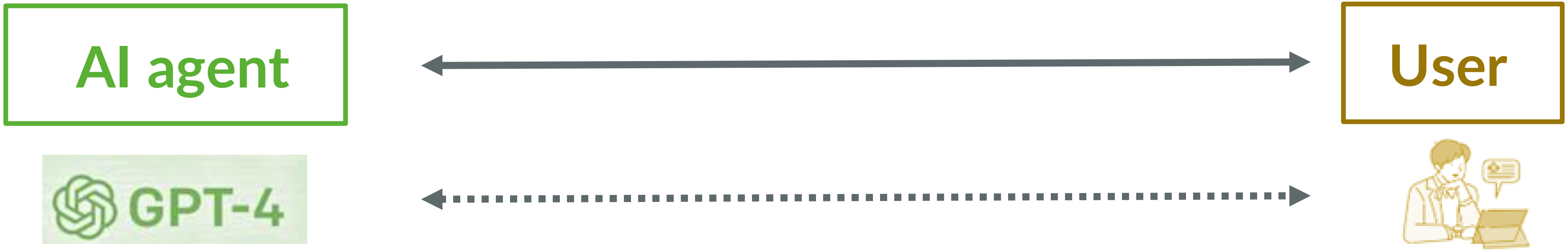

Motivation: User-AI Interactions

AI agent

User

GPT-4

YouTube

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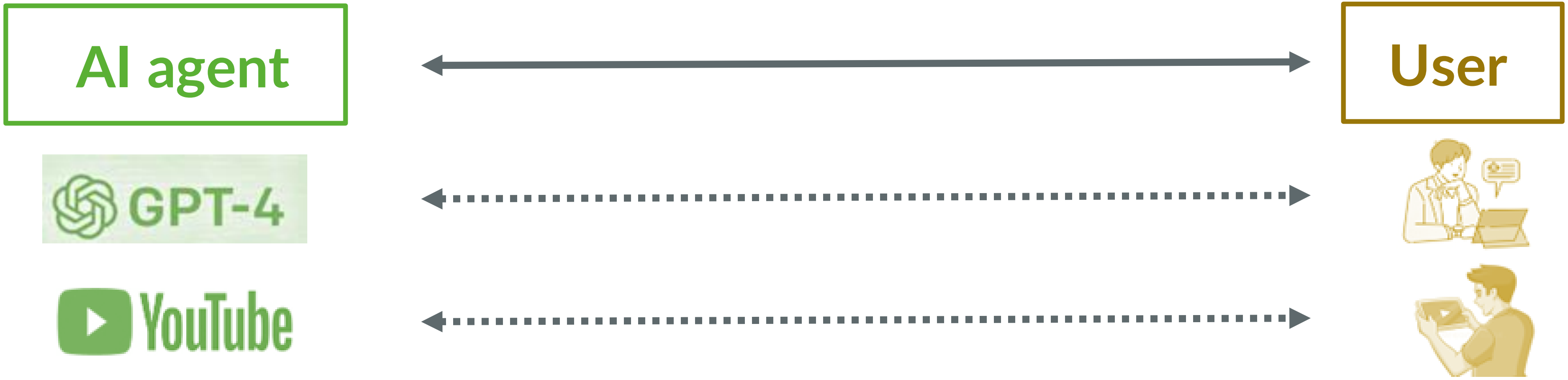
AI agent learns from repeatedly interacting with a user.

Motivation: User-AI Interactions

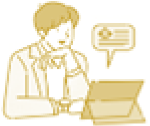


AI agent learns from repeatedly interacting with a user.

User learns from repeatedly interacting with the AI agent.

Motivation: User-AI Interactions

AI agent ⟷ User

GPT-4 

YouTube 

AI agent learns from repeatedly interacting with a user.

User learns from repeatedly interacting with the AI agent.

User-AI interactions are a form of multi-agent learning.

Our focus: Sequential, Misaligned, Decentralized Systems

User **(leader)**

GPT-4 Chatbot **(follower)**

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*User **(leader)***

GPT-4 *Chatbot **(follower)***

M **You**

Prompt

Sequential: User leads (w/ prompt)

Our focus: Sequential, Misaligned, Decentralized Systems

*User **(leader)*** ⟷ GPT-4 *Chatbot **(follower)***

You — Prompt

ChatGPT — Output

Sequential: User leads (w/ prompt); chatbot follows (w/ output).

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Misaligned: User utility (individual preferences) vs. Chatbot utility (e.g., societal preferences, implicit objective learned during training)

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You Prompt — **ChatGPT** Output — **You** Prompt — **ChatGPT** Output

Sequential: User leads (w/ prompt); chatbot follows (w/ output).

Misaligned: User utility (individual preferences) vs. Chatbot utility (e.g., societal preferences, implicit objective learned during training)

Decentralized: User + chatbot learn separately over a chat session.

Main question

We study **Stackelberg games with decentralized learning**, which are:

- **Sequential**: One agent leads, and the other agent follows.
- **Misaligned:** Agents have different utility functions.
- **Decentralized:** Agents learn separately from repeated interactions.

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- Implications for **each agent's cumulative utility over time?**

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- Implications for **each agent's cumulative utility over time?**
- Implications for **learning algorithm design**?

Overview of our contributions

Misalignment in agent utilities distorts the learning dynamics.

- When agents can be arbitrarily misaligned, we show the (full-information) Stackelberg equilibrium utilities are **unachievable**.

- We develop **relaxed benchmarks** for each agent's utility, and construct algorithms that perform well for both agents w.r.t. these benchmarks.

- When agents are partially aligned, we show the Stackelberg equilibrium utilities can be achieved.

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2. **Model of Stackelberg games with decentralized learning**
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Overview of our model

- ❑ **Sequential** (one agent goes first)
- ❑ **Misaligned** (agent utilities are not equal)
- ❑ **Decentralized** (agents learn separately)

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Stackelberg games: sequential, misaligned, **static** environments

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Our model: Stackelberg games with decentralized learning

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- ✓ **Sequential** (one agent goes first)
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- ✓ **Decentralized** (agents learn separately)

Stackelberg games: sequential, misaligned, **static** environments

Our model: Stackelberg games with decentralized learning

- Agents interact over T rounds.
- At every round, the leader goes first, and the follower goes second.
- Each agent observes their own **stochastic rewards**.

Recap of static Stackelberg games

	Action	Utility
Leader	$a \in A$	$u_1(a, b)$
Follower	$b \in B$	$u_2(a, b)$

Follower best-responds to leader: $b^*(a) = argmax_{b \in B} u_2(a, b)$

Leader **anticipates follower's actions** and best-responds:

$$a^* = argmax_{a \in A} u_1(a, b^*(a))$$

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Requires full knowledge of u_2

Leader **anticipates follower's actions** and best-responds:

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Requires full knowledge of u_1 and b^*

Stackelberg games with decentralized learning

Each agent **learns** how to select actions over T rounds.

Stackelberg games with decentralized learning

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We cast agent learning within the **stochastic multi-armed bandit framework**:

Leader

A = arms, u_1 = mean reward function

$\boldsymbol{ALG_1}$ **= bandit algorithm**

Follower

B = arms, u_2 = mean reward function

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Observes a_t & chooses b_t using $\boldsymbol{ALG_2}$

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Leader:

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Observe **stochastic reward** $u_1(a_t, b_t) + \eta_{1,t}$

Follower:

Observes a_t & chooses b_t using $\boldsymbol{ALG_2}$

Observe **stochastic reward** $u_2(a_t, b_t) + \eta_{2,t}$

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Cumulative utility: $\sum_{t=1}^{T} u_1(a_t, b_t)$

Cumulative utility: $\sum_{t=1}^{T} u_2(a_t, b_t)$

Related work

Learning in Stackelberg games with unknown utilities for both agents:

e.g., Camara, Hartline, Johnsen (2020), Bai, Jin, Wang, Xiong (2021), Gan, Han, Wu, Xu (2023), Haghtalab, Podimata, Yang (2023), Collina, Roth, Shao (2023), etc.

Broader literature on learning in Stackelberg games:

e.g., Letchford, Conitzer, Munagala (2009), Balcan, Blum, Haghtalab, Procaccia (2015), Braverman, Mao, Schneider, Weinberg (2018), Fiez, Chasnov, Ratliff (2019), Deng, Schneider, Sivan (2019), Zrnic, Mazumdar, Sastry, Jordan (2021), Kao, Wei, Subramanian (2022), Goktas, Zhao, Greenwald (2022), Haghtalab, Lykouris, Nietert, Wei (2022), Zhao, Zhu, Jiao, Jordan (2023), Brown, Schneider, Vodrahalli (2023), Guruganesh, Kolumbus, Schneider, Talgam-Cohen, Vasileios-Vlatakis-Gkaragkounis (2024), etc.

Interacting learners:

*e.g. Chayes, Immorlica, Jain, Etesami, Mahdian (2007), Chan, Hadfield-Menell, Srinivasa, Dragan (2010), Daskalakis, Deckelbaum, Kim (2011), Borgs, Agarwal, Luo, Neyshabur, Schapire (2017), Aridor, Mansour, Slivkins, Wu (2020), Zhuang, Hadfield-Menell (2020), **J.**, Jordan, Haghtalab (2023), etc.*

Our model: stochastic rewards, utility of **both** agents, **arbitrary** misalignment, decentralized learning

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Measuring each agent's regret

We study each agent i's expected pseudo-regret: $\mathrm{E}[\sum_{t=1}^{T} u_i(a_t, b_t)] - \alpha_i \cdot T$

Agent i's cumulative utility

Agent i's benchmark

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Agent i's cumulative utility

Agent i's benchmark

What benchmarks are appropriate for this environment?

Stackelberg benchmark is unachievable

Naïve benchmark: $\alpha_i^{orig} := u_i(a^*, b^*(a^*))$ (the Stackelberg equilibrium utility)

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	b_1	. b_2
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Due to misalignment, small errors by one agent can distort the other agent's utility.

Our error-tolerant benchmarks

Account for agent errors via **ϵ-approximate best-response sets**:

<u>**Definition (Error-tolerant benchmarks):**</u>

$\alpha_1^{tol} := \inf_{\epsilon \leq \gamma} \left(\max_{a \in A} \min_{b \in B_\epsilon(a)} u_1(a,b) + \epsilon \right)$ and $\alpha_2^{tol} := \inf_{\epsilon \leq \gamma} \left(\min_{a \in A_\epsilon} \max_{b \in B} u_2(a,b) + \epsilon \right)$

$$B_\epsilon(a) := \left\{ b \in B \mid u_2(a,b) \geq \max_{b' \in B} u_2(a,b') - \epsilon \right\}$$

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Example of error-tolerant benchmarks

$\alpha_1^{tol} := \inf_{\epsilon \leq \gamma} \left(\max_{a \in A} \min_{b \in B_\epsilon(a)} u_1(a,b) + \epsilon \right)$ and $\alpha_2^{tol} := \inf_{\epsilon \leq \gamma} \left(\min_{a \in A_\epsilon} \max_{b \in B} u_2(a,b) + \epsilon \right)$

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We take $\delta < \gamma = 0.05$ in this example.

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Green is $B_\delta(a)$ Pink is A_δ

$$\boldsymbol{(\alpha_1^{tol}, \alpha_2^{tol}) = (0.5 + \delta, \delta)}$$

$$\boldsymbol{(\alpha_1^{tol}, \alpha_2^{tol}) = (0.5, 3\delta)}$$

We take $\delta < \gamma = 0.05$ in this example.

Our algorithmic results

Goal: design algorithms that achieve sublinear regret for both agents

- Standard algorithms can incur **linear regret** w.r.t the error-tolerant benchmarks.
- We construct algorithms achieving $\tilde{O}\left(T^{\frac{2}{3}}\right)$ regret w.r.t error-tolerant benchmarks.
- Any algorithms incur $\Omega\left(T^{\frac{2}{3}}\right)$ regret for some agent w.r.t error-tolerant benchmarks.

- When agent utilities are partially aligned, we construct algorithms which achieve $\tilde{O}\left(T^{\frac{1}{2}}\right)$ regret w.r.t the **original Stackelberg benchmarks.**

Conclusion

Standard algorithms can incur linear regret

Proposition (Informal): Suppose both agents run $\mathrm{ExploreThenCommit}$. Then **both agents** can **incur linear regret** w.r.t. the error-tolerant benchmarks.

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Leader's estimated rewards during exploration:

- $0.5\left(u_1(a_1, b_1) + u_1(a_1, b_2)\right) = \mathbf{0.4}$ on a_1

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Proposition (Informal): Suppose both agents run ExploreThenCommit. Then **both agents** can **incur linear regret** w.r.t. the error-tolerant benchmarks.

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Key issue: the follower's exploration phase distorts the leader's learning

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Regret is sublinear for both players!

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To distinguish instances, need to explore a **very suboptimal arm** (a_1, b_2) **for the leader**

What if agents are partially aligned?

Suppose that the two agents agree over which outcomes are different.

$$L = \sup_{a,a',b,b'} \left(\frac{u_1(a,b) - u_1(a',b')}{u_2(a,b) - u_2(a',b')}, \frac{u_2(a,b) - u_2(a',b')}{u_1(a,b) - u_1(a',b')} \right)$$

We show that L is bounded => the original Stackelberg benchmarks are achievable.

Theorem (Informal): There exist algorithms such that both players achieve $\tilde{O}\left(L^2\sqrt{T|A||B|}\right)$ regret w.r.t. the **original Stackelberg benchmarks**.

Takeaway: Partial alignment makes learning easier and faster.

Conclusion

We study Stackelberg games with decentralized learning.

Main finding: misalignment in agent utilities distorts learning dynamics

- We showed that the Stackelberg equilibrium utilities are unachievable.
- We designed error-tolerant benchmarks to better capture learning dynamics.
- We constructed algorithms which achieve optimal $\widetilde{\Theta}\left(T^{\frac{2}{3}}\right)$ regret.
- We showed that partial alignment makes learning easier and faster.

Future directions: allow for greater flexibility in leader algorithm, study application-specific learning algorithms, characterize equilibria in the meta-game between agents, etc.