Regret Minimization with Performative Feedback

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joint work with

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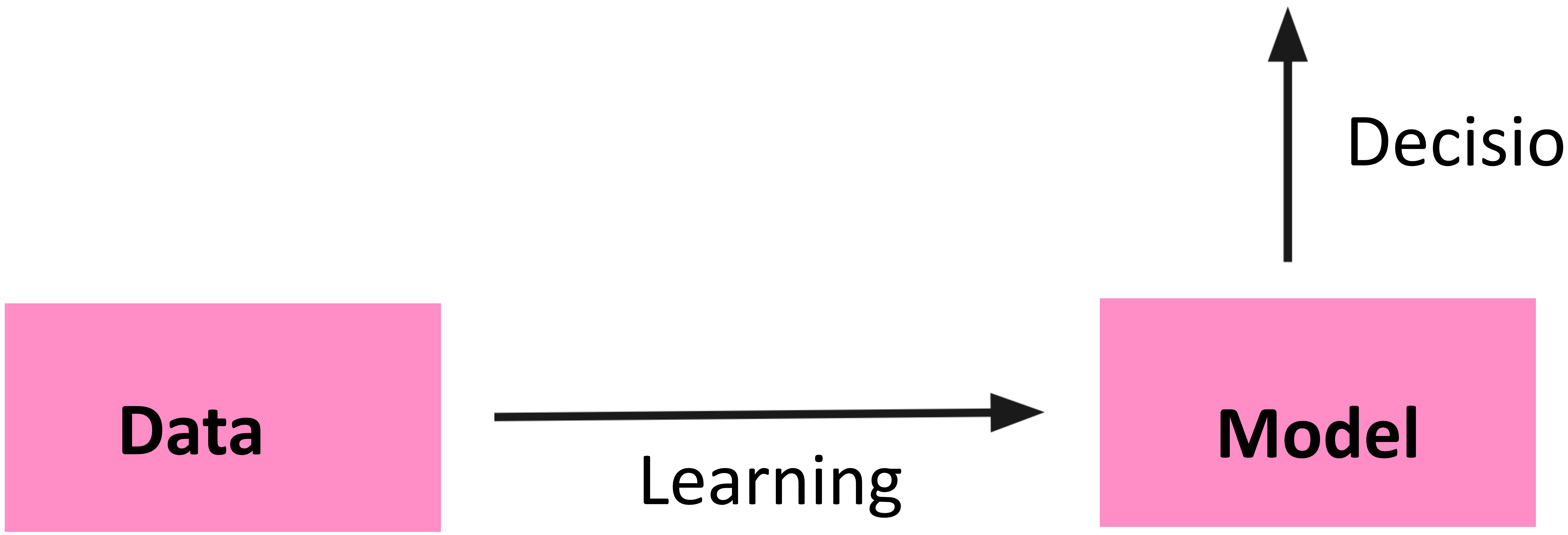
Static view of machine learning



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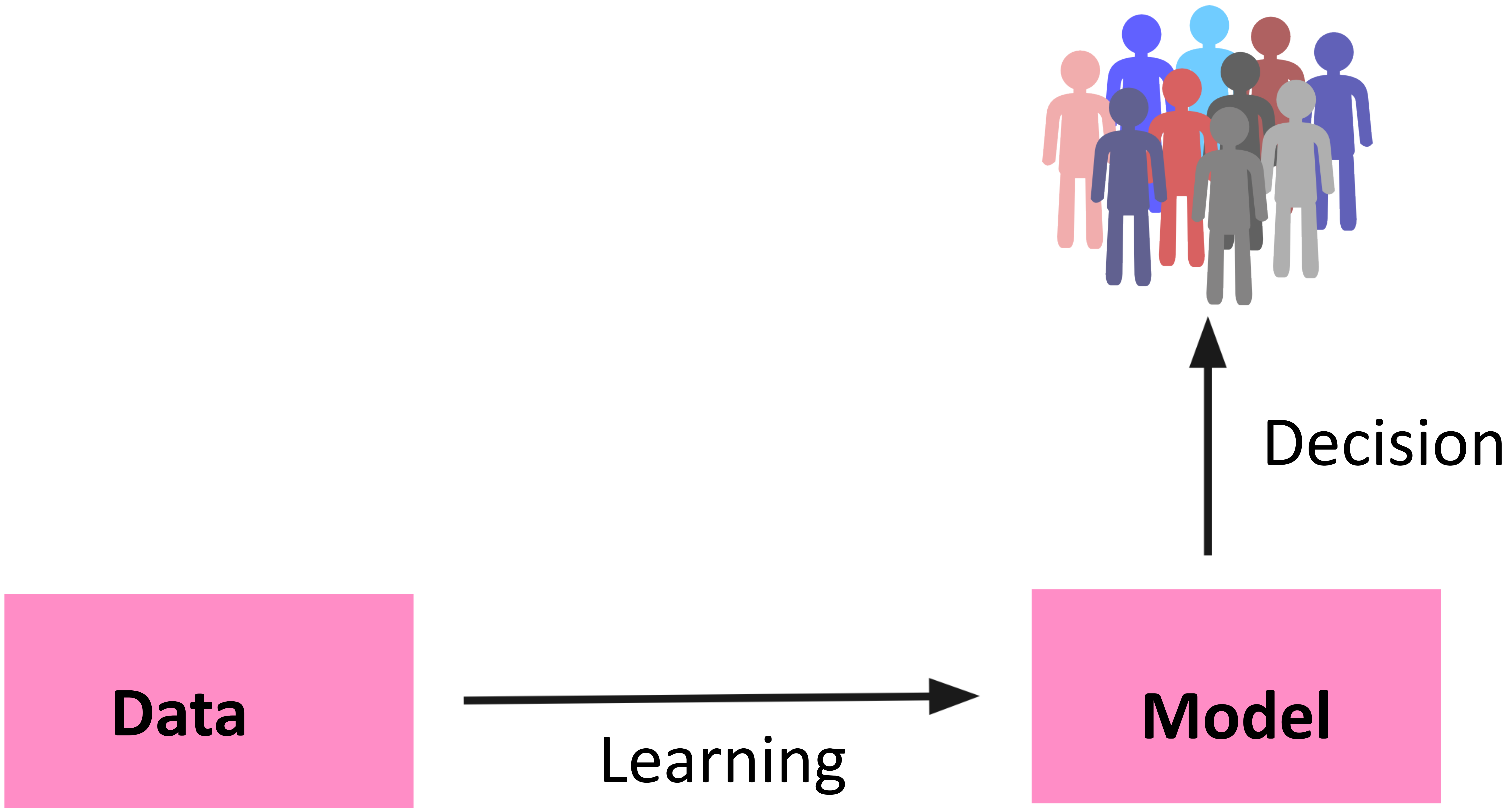
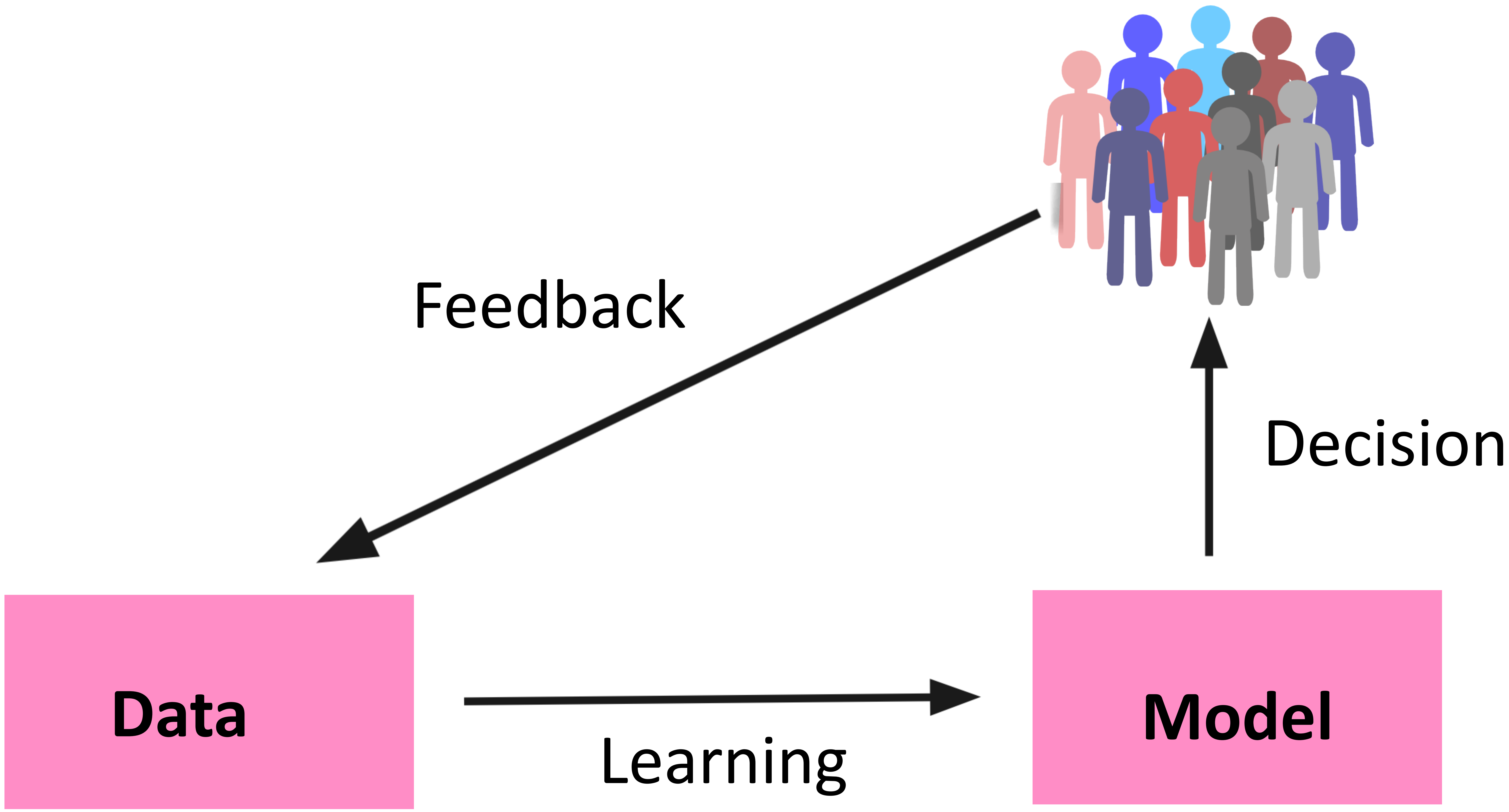


Machine learning with feedback effects



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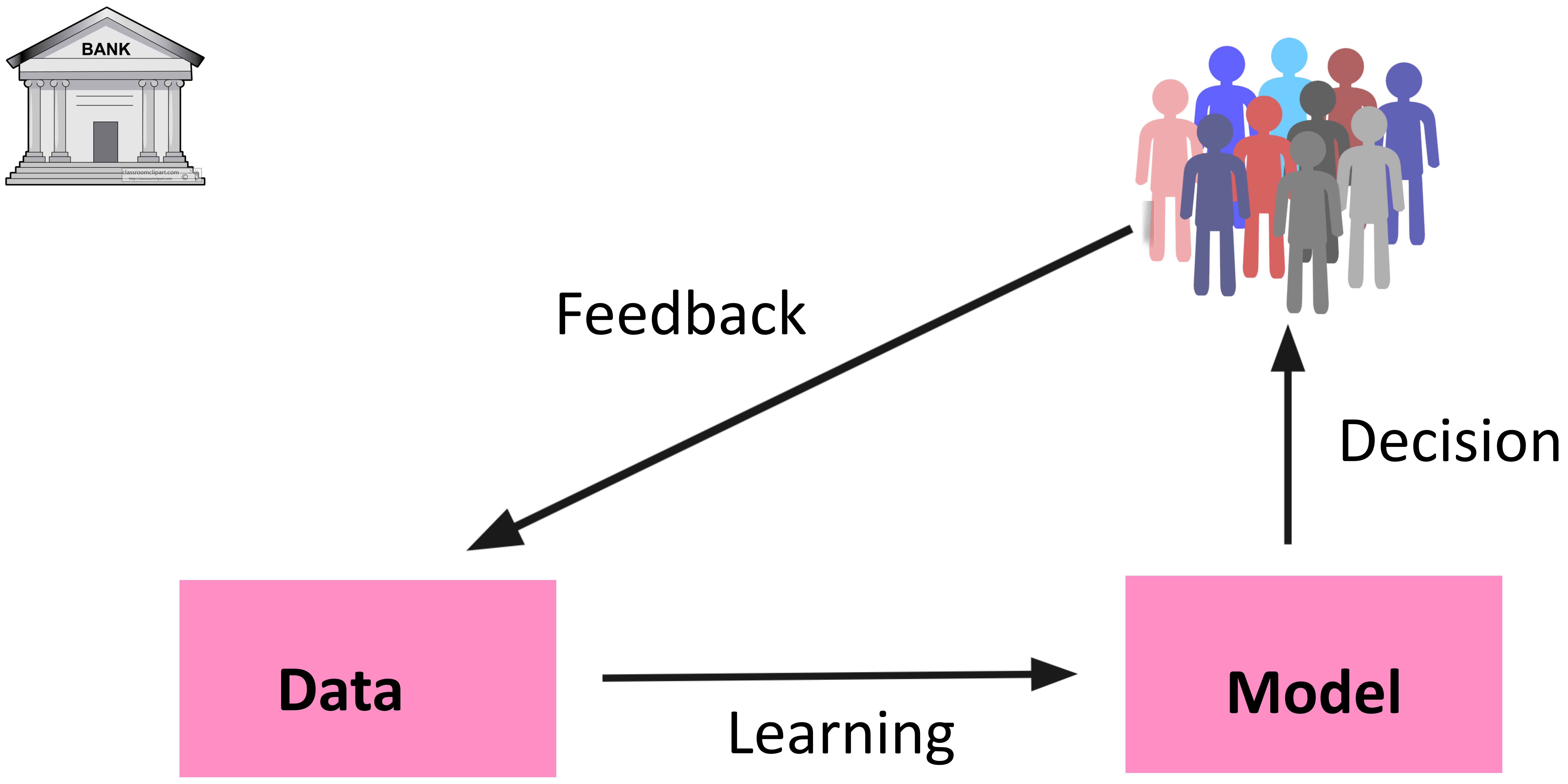

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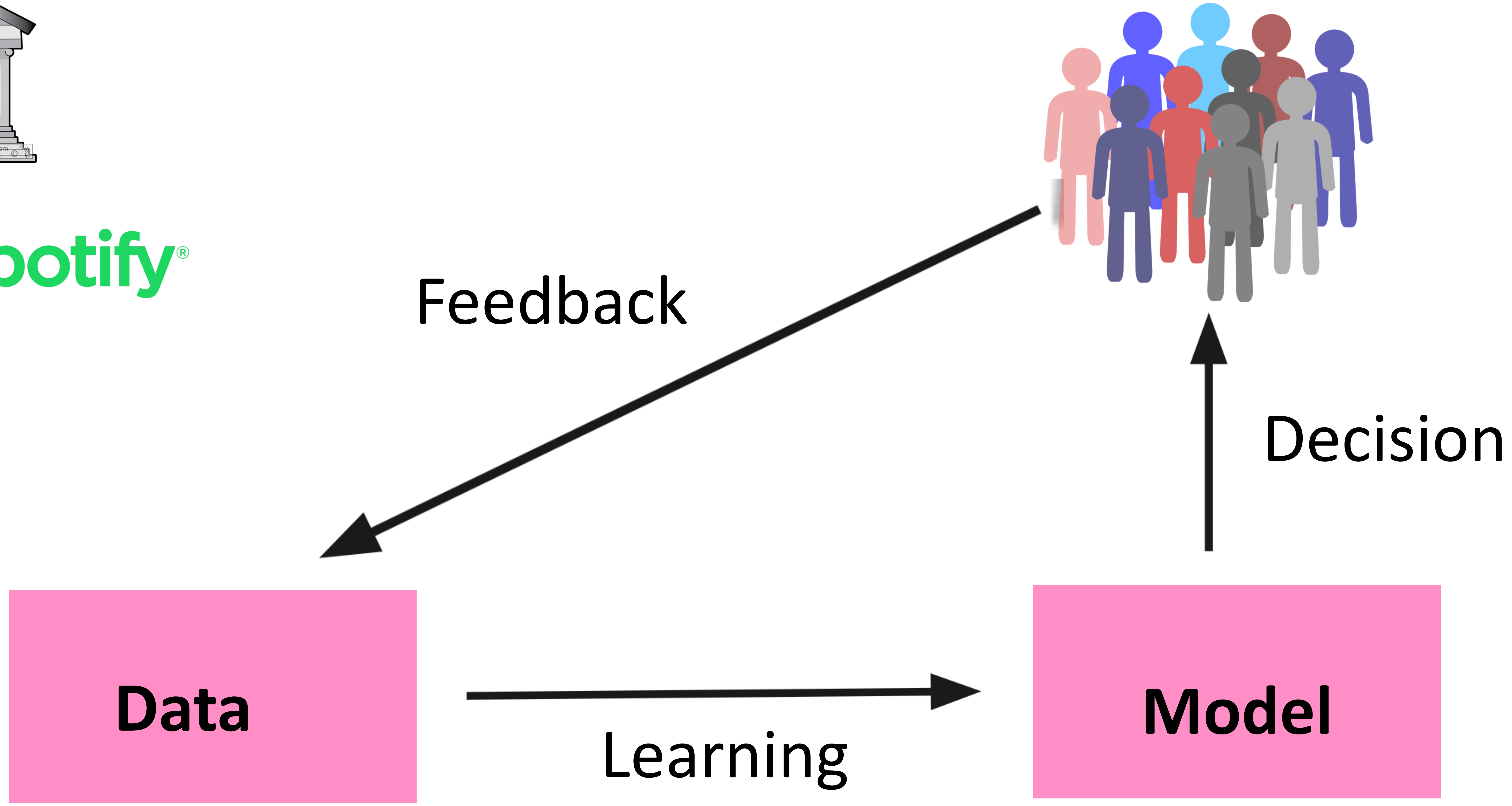
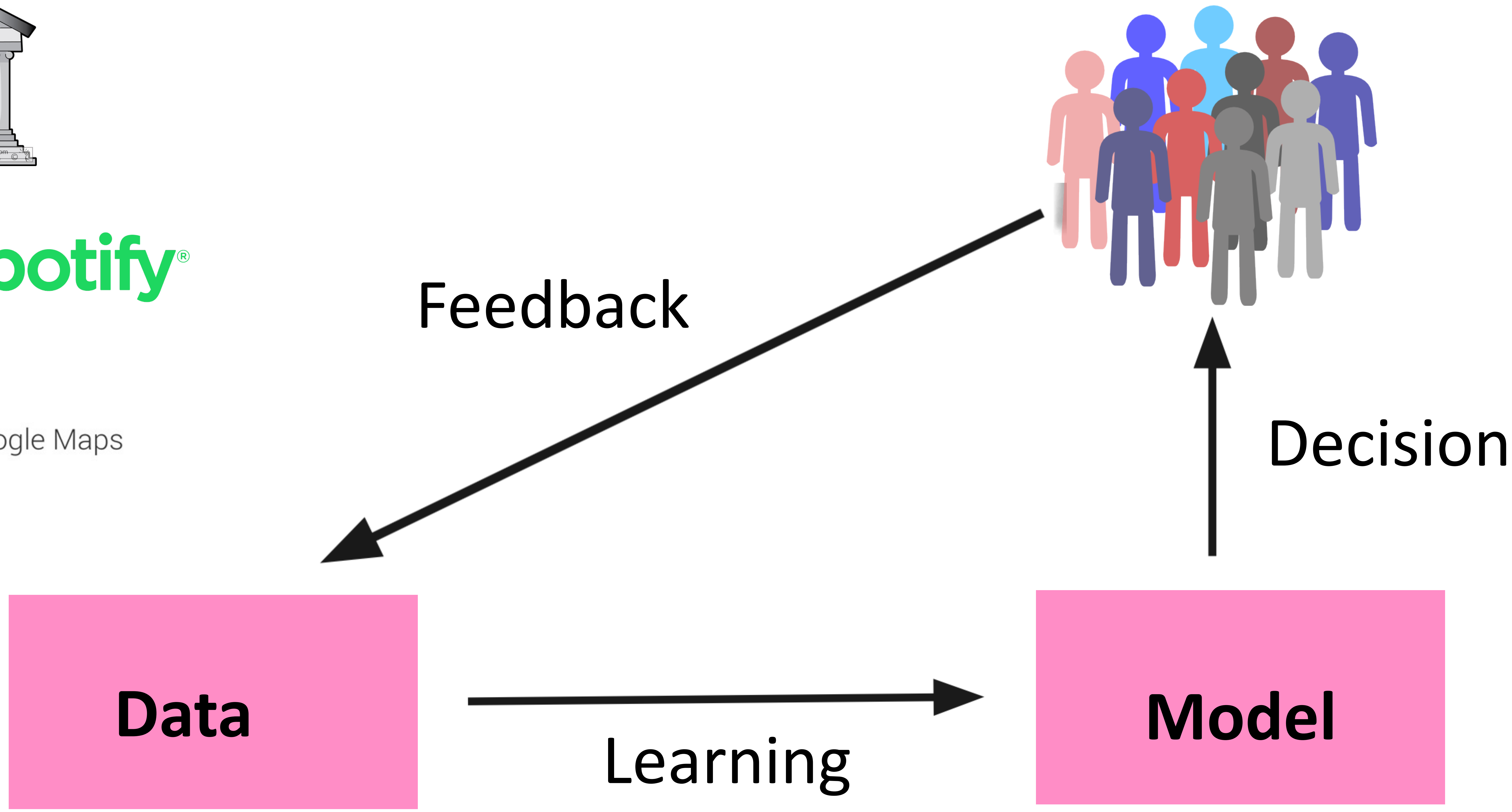


Machine learning with feedback effects

Spotify

Google Maps



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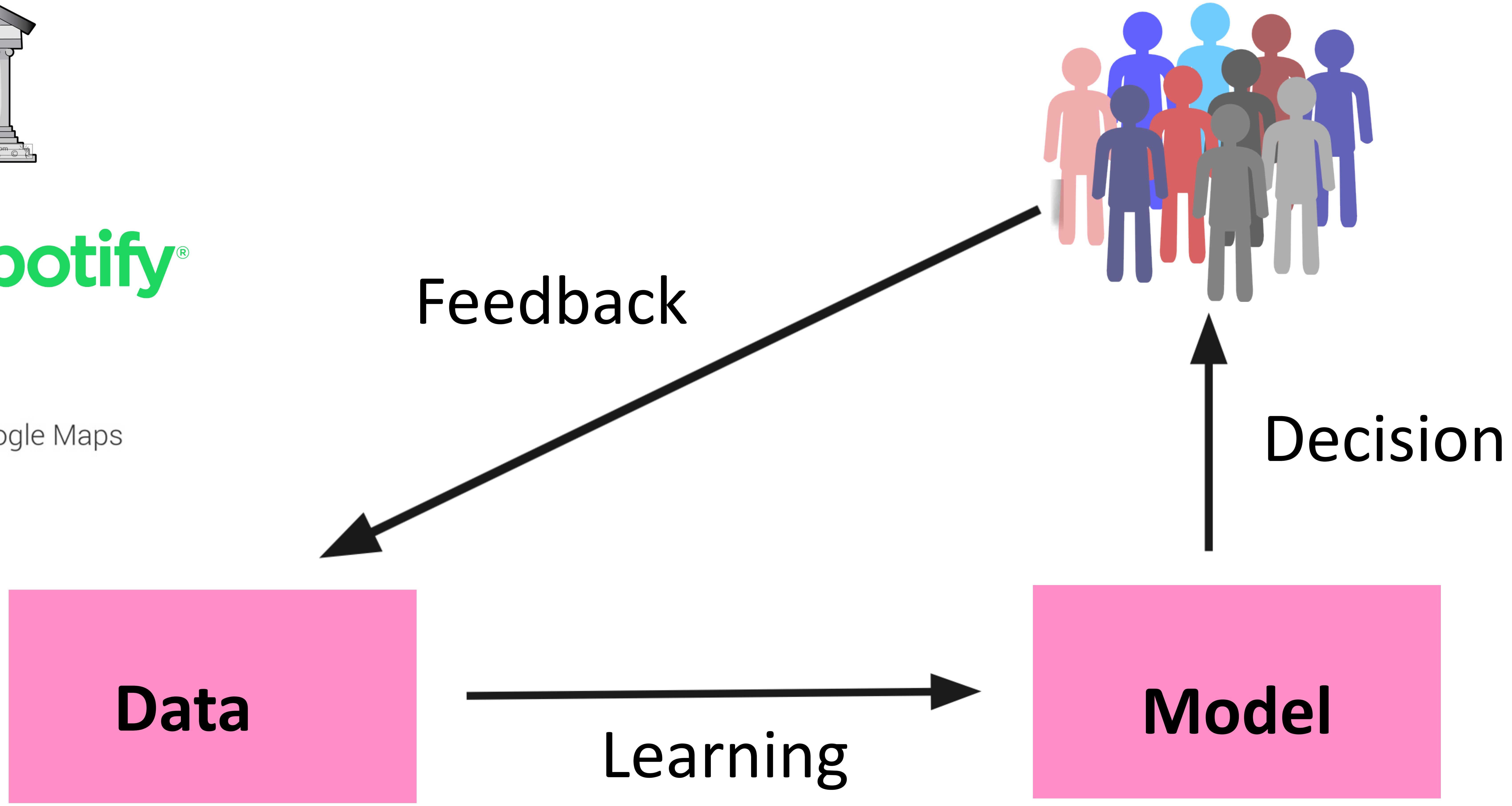

Our contribution: Learning algorithms that perform well in the presence of feedback effects

(Perdomo, Zrnic, Mendler-Dünner, Hardt, 2020)

Performative prediction

Typical supervised learning: data $Z = (X, Y)$ distributed according to a fixed distribution $\mathcal{D}$

The objective is to minimize *risk*: $\theta^* = \operatorname*{argmin}_{\theta} \mathrm{R}(\theta) \quad \mathrm{R}(\theta) = \mathop{\mathbb{E}}_{Z \sim \mathcal{D}} \ell(Z; \theta)$

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Performative prediction: model induces a distribution shift in the data distribution.

- Each model f_θ induces possibly distinct distribution $Z = (X, Y)$ over observations.
- We call $\mathcal{D}(\theta)$ the distribution map.

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The dependence on the model appears twice.

Our contributions

$$\theta^* = \operatorname*{argmin}_{\theta} \mathrm{PR}(\theta) \quad \mathrm{PR}(\theta) = \mathop{\mathbb{E}}_{Z\sim\mathcal{D}(\theta)} \ell(Z;\theta)$$

High-level approach: *Learn* distribution shifts through repeated model deployments

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High-level approach: *Learn* distribution shifts through repeated model deployments

1. We establish a connection between performative prediction and bandits.

   Optimization in performative prediction $\approx$ bandit problem with **richer feedback**

2. Under smoothness assumptions, we design an algorithm whose regret scales with the complexity of the distribution map and *not* with the complexity of the performative risk.

3. We extend our results to linear distribution maps.

Performative optimization as online learning

- The learner needs to deploy different θ to explore the induced distributions $\mathcal{D}(\theta)$

- Natural to evaluate online sequence of deployments $\theta_1, \ldots, \theta_T$ via **performative regret**:

$$\mathrm{Reg}(T) = \sum_{t=1}^{T} \Big(\mathbb{E}\mathrm{PR}(\theta_t) - \mathrm{PR}(\theta^*) \Big) \quad \theta^* = \operatorname*{argmin}_{\theta} \mathrm{PR}(\theta)$$

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"Baseline" bandits approach: Pull "arm" θ_t and observe reward $\widehat{\mathrm{PR}}(\theta_t)$.

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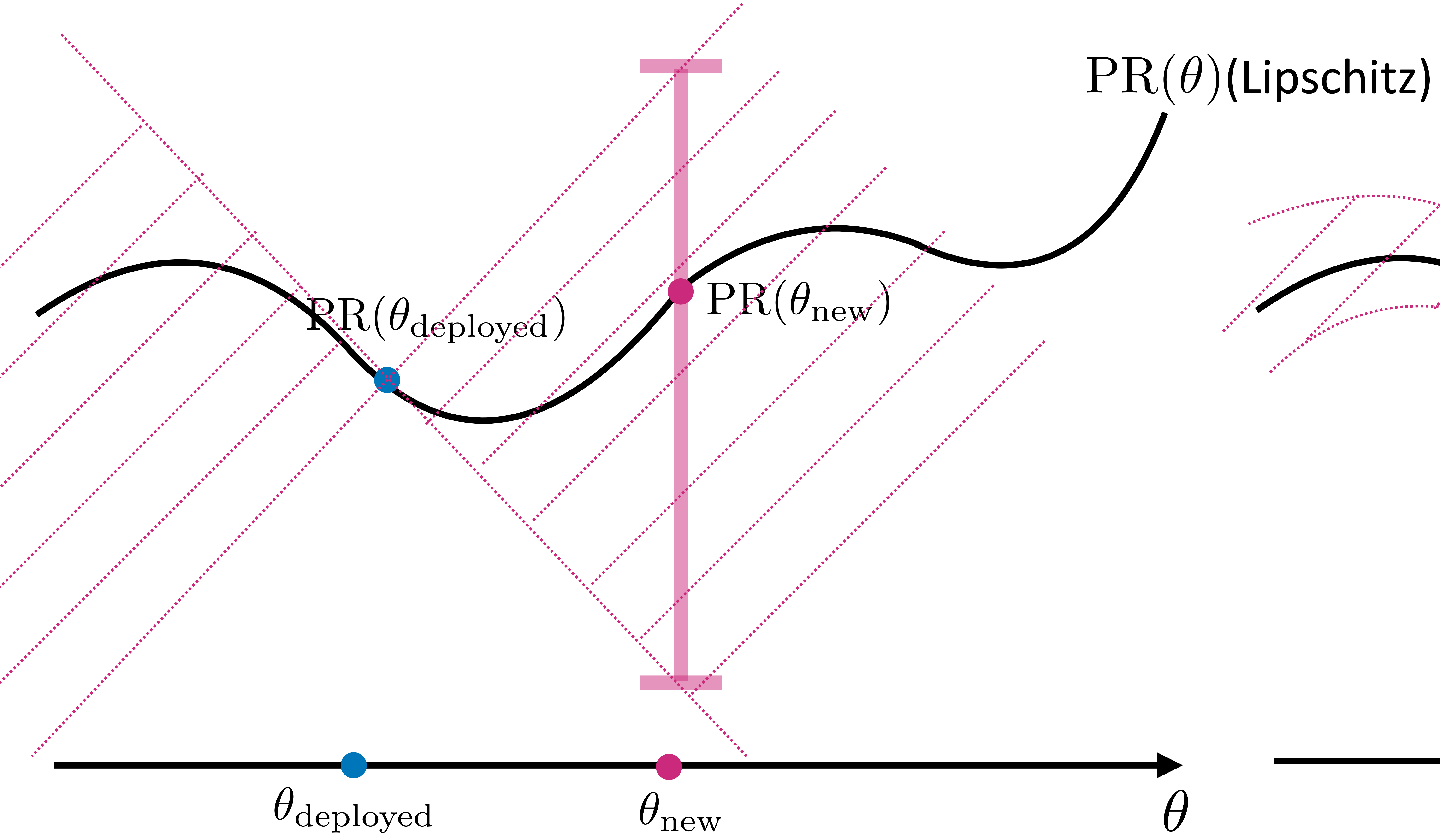
Main insight: performative settings exhibit **richer feedback** than bandit feedback.

- We observe **samples** from $\mathcal{D}(\theta_t)$, not just bandit feedback about performative risk.
- Can find θ^* with less exploration than bandit baselines.

Key insight: tighter confidence bounds

confidence bounds with bandit feedback:



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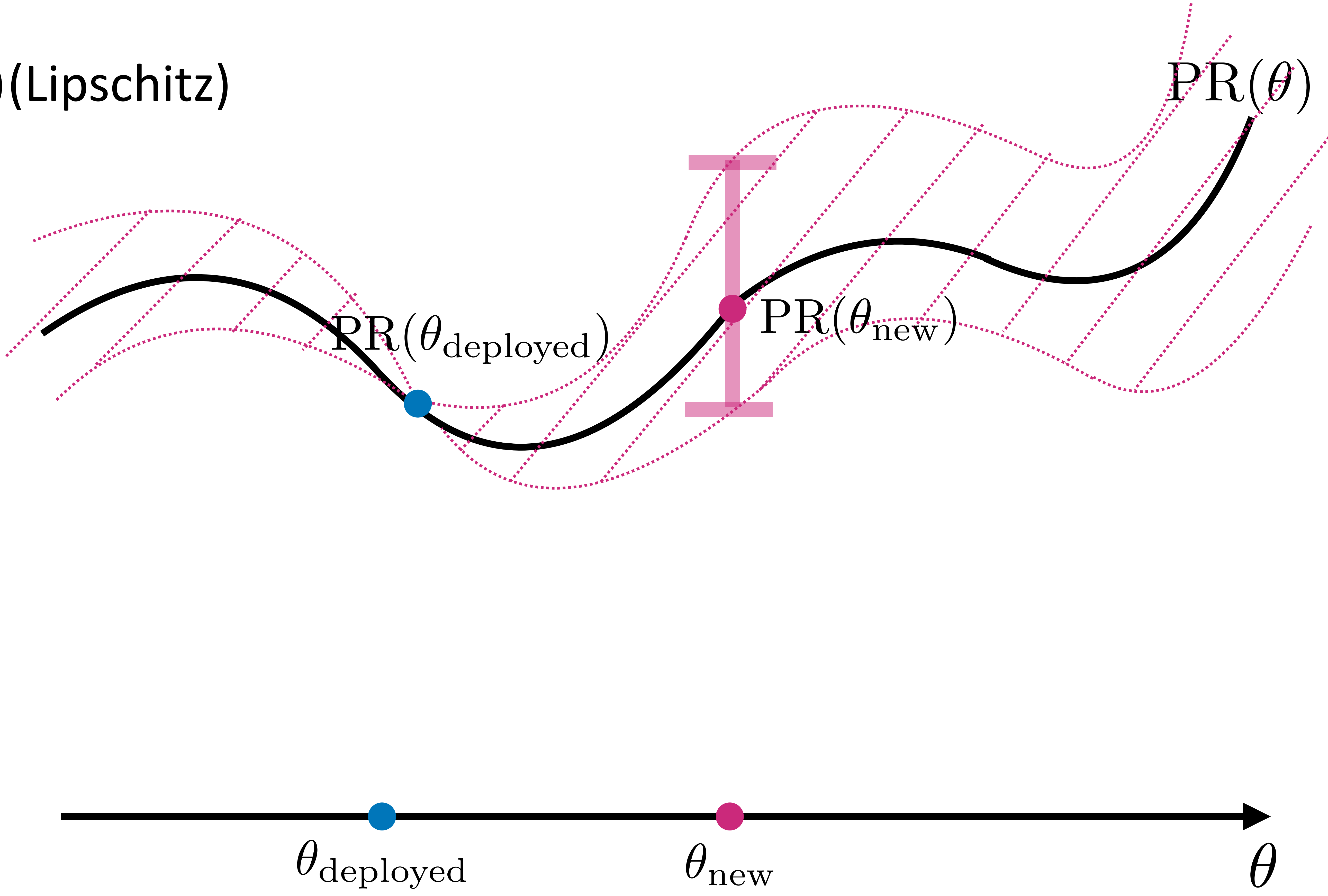

Assumption: distribution map is ϵ-Lipschitz in the model parameters

Regret bounds

Our bound

$$\text{Reg}(T) = \tilde{\mathcal{O}}\left(\sqrt{T} + T^{\frac{d+1}{d+2}} (L_z \epsilon)^{\frac{d}{d+2}}\right)$$

Lipschitz bandit baseline

$$\text{Reg}(T) = \tilde{\mathcal{O}}\left(T^{\frac{d'+1}{d'+2}} L^{\frac{d'}{d'+2}}\right)$$

$$L = (L_\theta + L_z \epsilon)$$

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Improvements over bandit baseline:

- our regret bound does not constrain loss $\ell(Z;\theta)$ as function of θ
- when performative effects are small, our bound becomes **dimension-independent**
- the zooming dimension with performative feedback is smaller, $d \leq d'$

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Takeaway: Regret scales only with the complexity of the distribution map and not that of the performative risk.

Conclusion and Future Work

- Deploying a model can induce a performative distribution shift on the population.
- Learner needs to deploy models online to find one $\approx$ with low induced risk
- Regret minimization with performative feedback $\approx$ bandit problem with richer feedback
- Performative feedback requires **less exploration** to find a good solution.

Future work: Leverage bandit tools for performative prediction more generally.

Thank you.